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Critical Dynamics Near the Superfluid Transition in He3-He4 Mixtures[1]

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ABSTRACT

The renormalization group(RG) functions for the critical dynamics at the λ-transition in He^3-He^4 Mixtures (model F') have been calculated in two loop order. Comparison is made with the hydrodynamic transport coefficients as function of concentration of He^3. Although general improvement is achieved with respect to an earlier calculation based on a combination of one loop and two loop terms in the field theoretic functions, some deviations remain in the mass diffusion coefficients. However the non universal initial values of the flow parameters take values more compatible with the physical expectation.

As a corollary we obtain the field theoretic functions of model F describing the critical dynamics of the supefluid transition in pure He^4. Comparing with experiments in pure He^4 improves the background value of the imaginary part of the time scale ratio between the order parameter and the energy.

KEY WORDS: critical dynamics; transport coefficients; superfluid transition; renormalization-group theory;

1. INTRODUCTION

Recently the field theoretic functions for the dynamical model (model F') that describes ^{3}He-^{4}He mixtures near the λ-transition have been calculated in two loop order [1]. Together with the two loop values for the amplitudes of the relevant vertex functions [2] the transport properties of these mixtures are now known completely up to two loop order. So far the critical dynamics has been treated [3, 4] in a combination of one and two loop terms. The terms with the static couplings to the secondary densities of entropy and concentration were taken in one loop order and all other dynamic terms (mode coupling terms) in two loop order. This approximation was used for a comparison with the temperature dependence of the three transport coefficients - thermal conductivity, thermal diffusion ratio and mass diffusion. The interrelation between the three coefficients allows a much better significant test of the theory. Apart from extrapolating the temperature dependence of the transport coefficients one may predict one of the three coefficients by fitting only the two other coefficients.

As a corollary of the field theoretic calculations, the field theoretic functions of model F [5] describing the critical dynamics of pure ^{4}He near the λ-transition could be checked and it turned out that the results for these functions presented in [5] differ from ours. We believe that our results are correct since the perturbational expansion fulfill the relation found between the correlation and response vertex function of the order parameter (OP) [1]. Moreover the field theoretic functions of [5] do not reduce to the correct model C functions [6]. It is therefore necessary to repeat the analysis of the thermal conductivity in order to find the non universal background values of the dynamical prameters. These may then be used for predicting other dynamical properties, like first sound attenuation [7, 8].

2. MIXTURES OF HELIUM4 and HELIUM3

The dynamical model for ^{3}He-^{4}He mixtures has been introduced by Siggia and Nelson [9]. It consists of three equations for the complex superfluid OP ψ ($n = 2$), and two linear combinations m_i of the entropy density and the concentration (see Eqs (2.17), (2.18) of [3]). They read

$$\begin{aligned}
\frac{\partial\psi}{\partial t} &= -2\Gamma\frac{\delta H}{\delta\psi^*} + i\psi\sum_i g_i\frac{\delta H}{\delta m_i} + \Theta_\psi \\
\frac{\partial\psi^*}{\partial t} &= -2\Gamma^*\frac{\delta H}{\delta\psi^+} - i\psi^*\sum_i g_i\frac{\delta H}{\delta m_i} + \Theta_\psi^* \\
\frac{\partial m_i}{\partial t} &= -\vec{\nabla}\vec{J}_i + \Theta_{m_i} \qquad i = 1, 2
\end{aligned}$$

with the currents

$$\begin{aligned}
\vec{J}_1 &= -\lambda\vec{\nabla}\frac{\delta H}{\delta m_1} - L\vec{\nabla}\frac{\delta H}{\delta m_2} - 2g_1\Im(\psi^*\vec{\nabla}\psi) \\
\vec{J}_2 &= -L\vec{\nabla}\frac{\delta H}{\delta m_1} - \mu\vec{\nabla}\frac{\delta H}{\delta m_2} - 2g_2\Im(\psi^*\vec{\nabla}\psi)
\end{aligned}$$

The non conserved OP relaxes with the complex relaxation coefficient $\Gamma = \Gamma' + i\Gamma''$, the kinetic coefficients λ, μ, and L correspond to thermal diffusion, mass diffusion and the thermo diffusion mode. Non vanishing Poisson brackets lead to reversible terms coupling the OP and the conserved densities with coefficients g_i. The stochastic forces Θ_i fulfill Einstein relations in order to reach the equilibrium described by the functional

$$H = \int d^dx \left\{\frac{1}{2}\tau|\psi|^2 + \frac{1}{2}|\nabla\psi|^2 + \frac{\tilde{u}}{4!}|\psi|^4 + \frac{1}{2}(m_1^2 + m_2^2) + \frac{1}{2}\gamma_2 m_2|\psi|^2\right\} \qquad (1)$$

where the static diagonalization and scaling of the densities has been performed.

Within the field theoretic renormalization group method one obtains by standard methods via the so called ζ-functions the flow equation of the dynamic parameters of the dynamic model. These parameters appear in the expressions for the physical dynamical quantities calculated from vertex functions. One example are the transport coefficients obtained in the hydrodynamic limit ($k\xi < 1$) from these vertex functions. The temperature dependence is determined by the flow equations of the dynamical parameters via a matching condition which relates the flow perameter to the correlation length ξ or the relative temperature distance $t = (T - T_\lambda)/T_\lambda$ respectively. The initial values necessary to solve the flow equations are the non universal quantities to be determined by comparison with experiment. Their values may appear in other dynamical quantities which then can be predicted.

The expression for the ζ-functions can be found in [1]. Inserting them into the flow equations given in [3] (see Eqs (3.56) there) the two loop temperature dependence of the dynamic parameters is known.

3. TRANSPORT COEFFICIENTS

The transport coefficients of He3-He4 mixtures have been calculated in one loop order in [3]. The higher loop terms of the thermal conductivity in pure He4 can be used to find the hydrodynamic transport coefficients of the mixture in this order. This feature results from the same coupling of all the conserved densities to the OP in statics as well as in dynamics. Inserting into Eqs (3.49) of [3] the result of the two loop terms calculated in [2] we have the complete two loop result at hand.

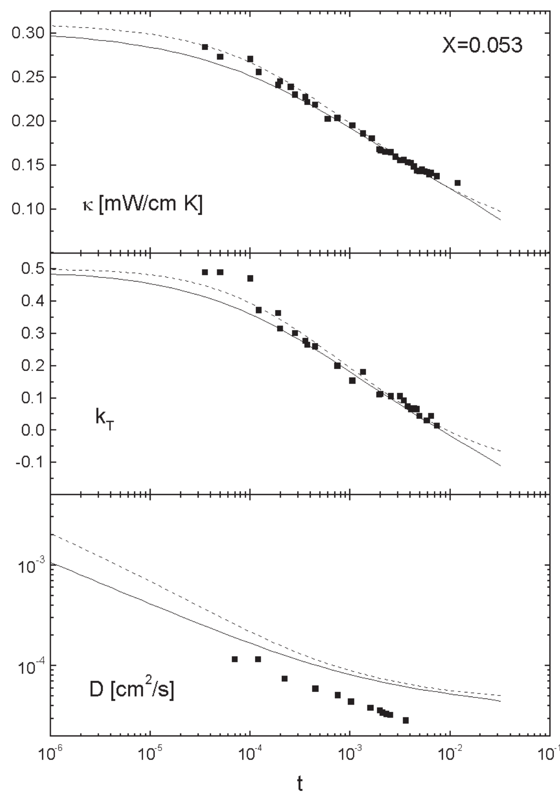

Figure 1: Temperature dependence ($t = T/(T - T_c)$) of the thermal conductivity κ, the thermal diffusion ratio k_T and the mass diffusion D in the mixture for the molar concentration X=0.053 (solid curves: complete two loop result; dashed curves: former incomplete theory; experimental data from [12, 13]). From a fit of the first two coefficients the third is predicted.

The main quantity to be calculated are the derivatives of vertex functions

$\hat{\Gamma}_{m_i\tilde{m}_j}$ by the wave vector modulus squared

$$\frac{\partial}{\partial k^2}\hat{\Gamma}_{m_1\tilde{m}_1} = \lambda\left(1-\frac{f_1 f_2}{4}(1-P(\{w_i\},\{f_i\}))\right) \tag{2}$$

$$\frac{\partial}{\partial k^2}\hat{\Gamma}_{m_1\tilde{m}_2} = \sqrt{\lambda\mu}\left(w_3-\frac{f_1^2}{4}(1-P(\{w_i\},\{f_i\}))\right) \tag{3}$$

$$\frac{\partial}{\partial k^2}\hat{\Gamma}_{m_2\tilde{m}_2} = \mu\left(1-\frac{f_2^2}{4}(1-P(\{w_i\},\{f_i\}))\right) \tag{4}$$

where the time scale ratios

$$w_1=\frac{\Gamma}{\lambda}\,, \qquad w_2=\frac{\Gamma}{\mu}\,, \qquad w_3=\frac{L}{\sqrt{\lambda\mu}}\,, \tag{5}$$

and the mode couplings

$$f_1=\frac{g_1}{\sqrt{\Gamma'\lambda}}\,, \qquad f_2=\frac{g_2}{\sqrt{\Gamma'\mu}} \tag{6}$$

The two loop perturbation contributions to the vertex functions are given by $P(\{w_i\},\{f_i\})$ and are related to the two loop contributions calculated in [2]. We have taken these expressions from [2] and they turn out to be rather small also in our case.

The comparison with experiment follows the same steps as explained in [4] we only list the experimental quantities used in this comparison. We need:

- the correlation length $\xi=\xi_0 t^{-\nu}$ at a certain pressure and concentration
- the λ-line derivatives of the entropy S, the molar concentration X and the difference of the chemical potentials of He4 and He3 Δ [10]

$$\left(\frac{\partial S}{\partial T}\right)_{P,\lambda}\,, \qquad \left(\frac{\partial X}{\partial T}\right)_{P,\lambda}\,, \qquad \left(\frac{\partial \Delta}{\partial T}\right)_{P,\lambda} \tag{7}$$

- the specific heat $C_{P,X}$ [11]

- the thermal conductivity κ at mass current zero, the mass difusion D and the thermal diffusion ratio k_T [12, 13]

An analysis of the experimental data has been repeated with the complete expressions known now. The λ-line derivatives have been taken from experiment and the flow of the fourth order coupling $u(\ell)$ and the coupling $\gamma(\ell)$ have been extracted from the experimental temperature dependence of the specific heat $C_{P,X}$, see [4].

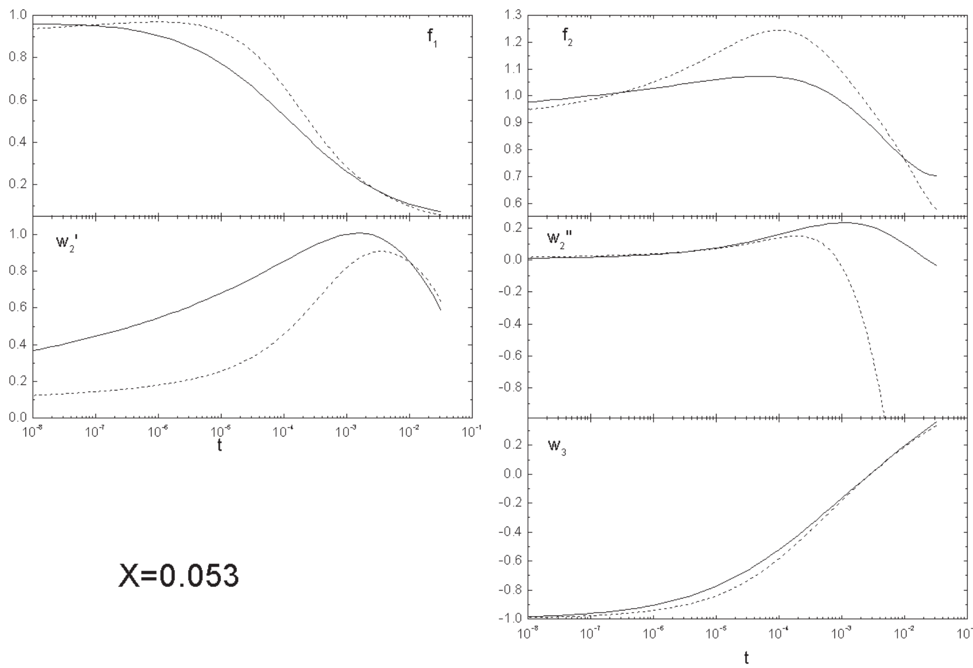

Figure 2: Comparison of the flow found from fits to the data as described in Fig. 1. Solid curve: complete two loop flow; dashed curve: former incomplete theory).

In Fig. 1 we show the comparison of the former results with the those now obtained for the lowest concentration ($X = 0.053$), fits for the other concentration can be found in [1]. The overall picture is an improvement of the prediction of the third transport coefficient, the mass diffusion. However substantial deviations remain. On the other hand the flows obtained from the complete and consistent two loop result is much more in accordance with the physical expectation (see Fig. 2). E.g. we expect the imaginary part w_2'' of the time ratio w_2 to be positive, which now is almost the case. It turns out that the behavior of the flow in this background region is very sensitive to the values of the experimental data within this region. Especially the behavior of the small and uncertain values of the thermal diffusion ratio k_T is important, e.g if it changes sign already or not.

4. PURE HELIUM4

Reducing the result for model F' by coupling only one conserved density to the OP we obtain the ζ-function of the OP for model F describing pure He4

$$\zeta_\Gamma = \mathcal{F}^2 + \frac{u^2}{9}\left(L_0 + x_1 L_1 - \frac{1}{2}\right) - \frac{2}{3}u\mathcal{F}a - \frac{1}{2}\mathcal{F}^2 b \tag{8}$$

with the definitions $\mathcal{F} = \mathcal{C} - i\mathcal{E}$, $\mathcal{C} = \sqrt{\frac{w}{1+w}}\,\gamma$ and $\mathcal{E} = \frac{F}{\sqrt{w(1+w)}}$. The OP relaxation $\Gamma = \Gamma' + i\Gamma''$ and the time ratio $w = \Gamma/\lambda = w' + iw''$ are complex, and the mode coupling is defined by $F = g/\lambda$. Note that in a and b which are defined as

$$a = \mathcal{C}(1 - x_1 L_1) + i\mathcal{E}x_- x_1 L_1 - \mathcal{F}L_0 \tag{9}$$

and

$$\begin{aligned} b = {} & \mathcal{C}^2(1 - 2x_1 L_1) + 2i\mathcal{C}\mathcal{E}(1 + x_- x_1 L_1) + \\ & \mathcal{E}^2\left(\left[x_+ + v + x_+^2\left(x_+^2 + 2v^2\right)\right]\frac{L_1}{x_+} - 3v\right) - 2\mathcal{F}^2 L_0 - \frac{\mathcal{F}^2}{1+w}\left[w + (1+2w)l^{(s)}\right] \end{aligned} \tag{10}$$

the couplings via $\mathcal{C}$, $\mathcal{E}$ and $\mathcal{F}$ are contained. Further we have $v = \frac{\Gamma}{\Gamma^*}$, $x_\pm = 1 \pm v$, $x_1 = 2 + v$ and the logarithmic terms

$$L_0 = 2\ln\frac{2}{1+v^{-1}}\ , \qquad L_1 = \ln\frac{\left(1+v^{-1}\right)^2}{1+2v^{-1}} \qquad l^{(s)} = \ln\frac{(1+w)^2}{1+2w} \tag{11}$$

For the ζ-function of the secondary density we get

$$\zeta_\lambda = \gamma^2 - \frac{F^2}{2w'}\Big(1 + \frac{1}{2}\Re[Q]\Big) \tag{12}$$

with

$$Q = \frac{w'}{w}\mathcal{F}\Big[\mathcal{F}\left(\frac{1}{2} + \ln\frac{1+w}{1+w^*}\right) + \sqrt{\frac{w^*}{w}}\mathcal{F}^*|1+w| - \Big(\frac{W}{w}\mathcal{C} + iw\mathcal{E}\Big)WL\Big] \tag{13}$$

and $W = w + w^* + ww^*$, $L = \ln(1 + W^{-1})$.

The difference to Dohm's result appear in the OP ζ function only

$$\begin{aligned}\zeta_\Gamma^{(Dohm)} - \zeta_\Gamma \;\; &= \;\; \frac{\gamma^2 D^2 x_-}{4w^2 x_+(1+w)}L_0 - \frac{w\gamma D F^2}{w^* x_+ (1+w)^2} \\ &+ \;\; \frac{D}{2(1+w)^2}\left[\frac{ww^* - w - w^*}{(w^*)^2 x_+}\gamma^3 - 2\frac{x_-}{w^2}DF^2\right]L_1\end{aligned} \tag{14}$$

The difference goes to zero in the asymptotic limit but contributes substantially in the physical accessible region to the flow.

The changes in the ζ_Γ-function effect the flow equations which govern the non asymptotic temperature dependence of the transport coefficients.

We also have compared our results for the thermal conductivity $\lambda_T(t)$ (which is also obtained in the limit $\lim_{X\to 0}\kappa(t,X) = \lambda(T)$ in mixtures) with the experimental data at saturated vapor pressure (SVP) in pure ^{4}He, respective with the effective amplitude $R_\lambda = \lambda_T/\sqrt{\xi g_0^2 k_B C_P}$ (see Fig. 2), with C_P the temperature dependent specific heat, g_0 the unrenormalized mode coupling and k_B the Boltzman constant. The background parameter for the renormalized imaginary part

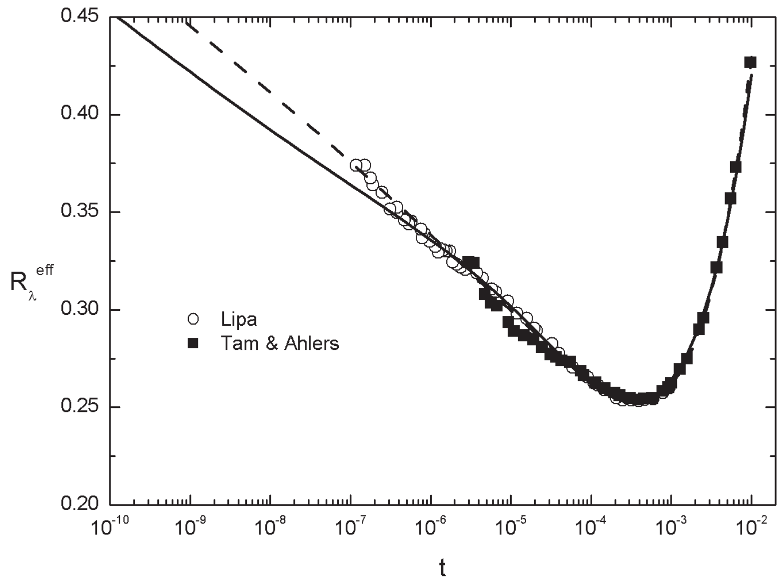

Figure 3: Fit and extrapolation (full curve) of the amplitude R_λ at SVP of the thermal conductivity in the range 10^{-3} to 10^{-6} to the data of [14] and above 10^{-3} to the data of [15]. Dashed curve fitted and extrapolated with the flow of [5].

of the time ratio w found from a fit with the correct flow equations is now of the expected size $w'' \approx 0.3$ instead of $w''_{Dohm} \approx 0.8$. In Ref. [16] the unrenormalized value was shown to be approximately $w''_0 \approx 0.21$.

Similar fits have been performed for higher pressures (see Fig. 4). However the imaginary part becomes smaller for higher pressures and reaches a value slightly smaller than zero ($w'' = -0.055$). Starting from the saturated vapor pressure $P_0 = 0.05$ bar one may fix the values of w'' at a higher pressures P by

$$w''_P = w''_{P_0} \frac{V_\lambda(P_0) C_P(P)}{V_\lambda(P) C_P(P_0)} \tag{15}$$

according to the relation derived by Dohm [16] for the *unrenormalized* param-

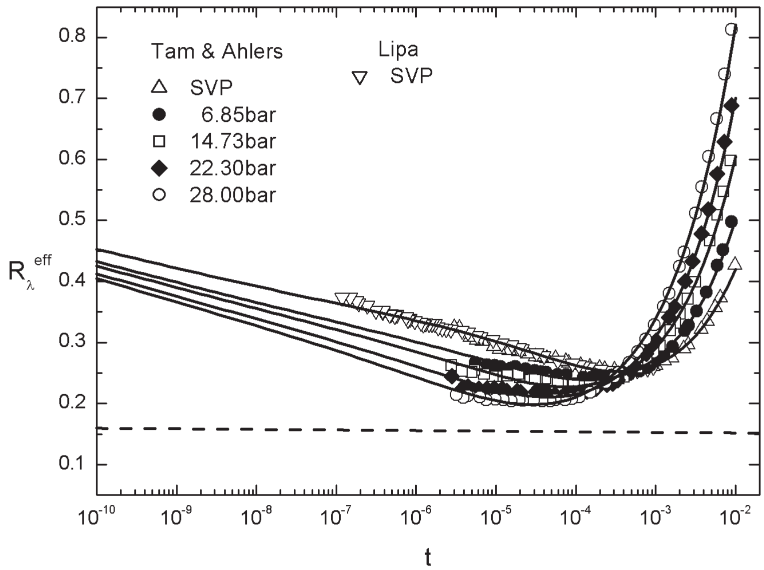

Figure 4: Fits (full curves) of the amplitude R_λ at different pressures. For comparison the *asymptotic* slope of the amplitude ratio is shown (dashed line), this slope is due to the fact that the non scaling fixed point $w^\star = 0$ is the stable one.

eters. This leads to less agreement with the data at the higher pressures most prominently at 28.0 bar.

Since in the two loop approximation the non scaling fixed point ($w^\star = 0$) is the stable one, an asymptotic powerlaw $R_\lambda \sim t^{-\omega_{corr}/\nu}$ with the correction to scaling exponent $\omega_{corr} = 0.08$ [5] is predicted. This is a much flatter increase approaching T_λ than obtained in the extrapolation region $10^{-10} \leq t \leq 10^{-7}$. Taking this into account it is surprising that the highest pressure data already show an almost flat behavior, whereas our calculation shows a more pronounced minimum for R_λ

5. CONCLUSION

Dynamical critical effects near the superfluid phase transition can only be compared within the non asymptotic renormalization theory in two loop order. An extension of this analysis to first sound phenomena should follow in order to get an overall picture of non asymptotic effects. On the experimental side it would be desirable for pure He4 to have more thermal conductivity data near T_λ at higher pressures especially for 28 bar in order to compare with the theoretical prediction, which indicates a sharper minimum in the data and a steeper increase. In general the two loop flow is more sensitive to the experimental data than the flow of the former incomplete theory. E.g. in mixtures they depend strongly on the background behavior of the thermo diffusion coefficient k_T, which then has a strong impact on the plateau value of the finite thermal conductivity κ.

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